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Exact solutions of non-autonomous quantum systems with semisimple Lie algebraic structure

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Abstract. If a non-autonomous quantum system has an semisimple Lie algebraic structure and its Hamiltonian can be treated as a linear function of the generators of a semisimple Lie group, we show a method for finding a set of gauge transformations that transform the Hamiltonian to a linear function of Cartan operators. The exact solutions of the equations of motion, as well as a set of time-dependent invariant operators which commute with each other, are obtained by the inverse gauge transformations. An SU(3) model serves as an illustration.

When considering the influence of the outside environment, the Hamiltonian of a quantum system often assumes a time-dependent form. Many important non-autonomous quantum systems are found to have algebraic structures and the Hamiltonian of the system can be treated as a linear function of the generators of a Lie group. For instance, the quantum motion of a particle in a Paul trap has an $\mathrm{SU}(1,1)$ structure [1, 2], and the Hamiltonian describing the controlling of the particle spin polarization in beam dynamics possesses an SU(2) structure [3]. Recently, the two-level density-dependent multiphoton Jaynes–Cummings models [4] has been shown to have an SU(2) structure [5]. Furthermore, some nonlinear quantum systems can be transformed into a linear system. For instance, the Hamiltonian describing an electron in a hydrogen atom can be transformed into a linear function of the generators of a U(4) group [6, 7].

There are many methods for finding solutions of a linear quantum system [8–12]. The problem with all these methods is that the procedures are very complicated for high-dimensional algebras. The purpose of this paper is to show an effective method for obtaining exact solutions of non-autonomous quantum systems with semi-simple Lie algebraic structure by means of the algebraic dynamics [1].

For some special cases, such as SU(2), $\mathrm{SU}(1,1)$ and h(4) structures, the solutions have already been worked out by the algebraic dynamics [1, 3]. The main procedure is to find a gauge transformation that transforms the Hamiltonian to a linear function of Cartan operators which commute with each other. In general, this method can be used for any linear systems. However, for semi-simple Lie algebraic structure the solution can easily be obtained.

The Hamiltonian of a linear non-autononous quantum system can be written as

$$H(t) = p_1(t)X_1 + \cdots + p_N(t)X_N \tag{1}$$

where the $p_i(t)$, $i = 1, \ldots, N$, are functions of time t and $\{X_1, \ldots, X_N\}$ form a basis of a Lie algebra G of dimension N. If G is semisimple, we can find a basis in Cartan standard form [13]:

$$\{H_i, E_\alpha, E_{-\alpha} | i = 1, \ldots, l; \alpha = 1, \ldots, M\} \tag{2}$$

$$\begin{aligned} &[H_i, H_j] = 0 \\ &[H_i, E_\alpha] = \alpha_i E_\alpha \\ &[E_\alpha, E_{-\alpha}] = \sum_{i=1}^{l} \alpha^i H_i \\ &[E_\alpha, E_\beta] = N_{\alpha,\beta} E_{\alpha+\beta}. \end{aligned} \tag{3}$$

Here l is the rank of the semisimple Lie algebra G, and $M = (N - l)/2$ is the number of raising (or lowering) operators. The root of the raising operator E_α is positive, and the root of the lowering operator $E_{-\alpha}$ is negative. In this basis the Hamiltonian has the form

$$H = \sum_{i=1}^{l} a_i(t) H_i + \sum_{\alpha=1}^{M} (b_i(t) E_\alpha + c_i(t) E_{-\alpha}). \tag{4}$$

To obtain solutions of the Schrödinger equation

$$\mathrm{i}\frac{\partial}{\partial t} |\psi(t)\rangle = H |\psi(t)\rangle \tag{5}$$

we make a gauge transformation

$$H(t) \to H'(t) = U_g H U_g^{-1} + \mathrm{i}\frac{\partial U_g}{\partial t} U_g^{-1} \tag{6}$$

$$|\psi(t)\rangle \to |\psi'(t)\rangle = U_g |\psi(t)\rangle \tag{7}$$

where $U_g(t)$ is an time-dependent operator with inverse $U_g^{-1}(t)$. It is easy to see that $|\psi'(t)\rangle$ satisfies the equation

$$\mathrm{i}\frac{\partial}{\partial t} |\psi'(t)\rangle = H' |\psi'(t)\rangle. \tag{8}$$

Our aim is to choose a gauge transformation that transforms H into a linear combination of the Cartan operators

$$H'(t) = d_1(t) H_1 + \cdots + d_l(t) H_l. \tag{9}$$

To this end, we find that $U_g(t)$ can be chosen as $2M$ consecutive gauge transformations in the form

$$U_g(t) = U_2(t) U_1(t) \tag{10}$$

where U_1 and U_2 are M successive gauge transformations,

$$U_1(t) = \exp(\mathrm{i} f_M(t) E_M) \cdots \exp(\mathrm{i} f_1(t) E_1) \tag{11}$$

$$U_2(t) = \exp(\mathrm{i} g_M(t) E_{-M}) \cdots \exp(\mathrm{i} g_1(t) E_{-1}). \tag{12}$$

The time-dependent parameters $f_i(t)$ and $g_i(t)$ are determined by equations (21) and (24) with the initial condition that at time $t = 0$ $f_i(t)$ and $g_i(t)$ satisfy

$$f_i(0) = g_i(0) = 0 \qquad i = 1, 2, \ldots, M. \tag{13}$$

The sequence of $E_1, E_2, \ldots, E_M$ is arranged in increasing order of their corresponding roots.

The f_i, $(i = 1, \ldots, M)$, are obtained by the requirement that after the gauge transformation U_1, the coefficients of $E_1, \ldots, E_M$ vanish. From this requirement, we get a set of differential equations by substituting $U_1(t)$ of (11) in (6):

$$\dot{f}_i = F_i\left(a_1, \ldots, a_l, b_1, \ldots, b_M, c_1, \ldots, c_M, f_1, \ldots, f_M, \dot{f}_1, \ldots, \dot{f}_{i-1}\right) \tag{14}$$

where $i = 1, \ldots, M$, and the right-hand side of the above equation is a polynomial of its variables. These equations can easily be obtained once we note the facts

$$e^{fH_i} E_\alpha e^{-fH_i} = e^{f\alpha_i} E_\alpha \tag{15}$$

$$e^{fE_\alpha} H_i e^{-fE_\alpha} = H_i - f\alpha_i E_\alpha \tag{16}$$

$$e^{fE_\alpha} E_{-\alpha} e^{-fE_\alpha} = E_{-\alpha} + f\sum_{i=1}^{l} \alpha^i H_i - \frac{f^2}{2}\left(\sum_{i=1}^{l} \alpha^i \alpha_i\right) E_\alpha \tag{17}$$

$$e^{fE_\alpha} E_\beta e^{-fE_\alpha} = \sum_{k=0}^{K} \frac{1}{k!} N_{\alpha,\beta} N_{\alpha,\alpha+\beta} \cdots N_{\alpha,(k-1)\alpha+\beta} E_{k\alpha+\beta}. \tag{18}$$

Here $\beta \neq \alpha$, $K \leqslant 3$ and K is determined by the condition that $k\alpha + \beta$ is a root, but $(K+1)\alpha + \beta$ is not. If the roots of E_α and E_β are both positive, then the root of $E_{k\alpha+\beta}$, which is also positive, is larger than that of E_α. Thus $\dot{f}_{i+1}, \dot{f}_{i+2}, \ldots$ do not appear in the coefficient of E_i. That is why the right-hand side of equation (14) only contains $\dot{f}_1, \dot{f}_2, \ldots, \dot{f}_{i-1}$. After gauge transformation $U_1(t)$, the Hamiltonian becomes

$$H^{(1)}(t) = \sum_{i=1}^{l} a'_i(t) H_i + \sum_{\alpha=1}^{M} \left[b'_\alpha(t) E_\alpha + c'_\alpha(t) E_{-\alpha}\right] \tag{19}$$

where $b'_\alpha(t)$ reads

$$b'_\alpha(t) = -\dot{f}_i + F_i(a_1, \ldots, a_l, b_1, \ldots, b_M, c_1, \ldots, c_M, f_1, \ldots, f_M, \dot{f}_1, \ldots, \dot{f}_{i-1}). \tag{20}$$

If we let $b'_\alpha(t) = 0$, we obtain equations (14).

By inserting the right-hand side of $\dot{f}_1, \dot{f}_2, \ldots, \dot{f}_{i-1}$, of equations (14) in the right-hand side of $\dot{f}_i$, the above set of differential equations can be transformed into standard form:

$$\dot{f}_i(t) = F_i(a_1, \ldots, a_l, b_1, \ldots, b_M, c_1, \ldots, c_M, f_1, \ldots, f_M). \tag{21}$$

Here F_i is still a polynomial; thus for the initial condition (13), the above equations have a unique set of solutions $f_1(t), f_2(t), \ldots, f_M(t)$.

Now, the Hamiltonian becomes

$$H^{(1)}(t) = \sum_{i=1}^{l} a'_i(t) H_i + \sum_{\alpha=1}^{M} c'_\alpha(t) E_{-\alpha}. \tag{22}$$

The gauge transformation $U_2(t)$ transforms $H^{(1)}(t)$ into

$$H^{(2)}(t) = \sum_{i=1}^{l} a''_i(t) H_i + \sum_{\alpha=1}^{M} c''_\alpha(t) E_{-\alpha}. \tag{23}$$

Note that the E_α, $(\alpha = 1, \ldots, M)$, do not appear in the above equations. Similarly, we obtain the $g_i(t)$, $(i = 1, \ldots, M)$, by the requirement that the gauge transformation $U_2(t)$

transforms $H^{(1)}(t)$ into the form (9). By means of a similar argument, we find that the $g_i(t)$ satisfy the differential equations

$$\dot{g}_i = G_i(a'_1, \ldots, a'_l, c'_1, \ldots, c'_M, g_1, \ldots, g_M). \tag{24}$$

Here G_i, $(i = 1, \ldots, M)$ is a polynomial of its variables. There exist a unique set of solutions that satisfy the initial condition (13). Note that the coefficient of H_i remains $a'_i(t)$ after the gauge transformation U_2:

$$d_i(t) = a'_i(t) = a''(t). \tag{25}$$

After gauge transformation, the Schrödinger equation becomes

$$\mathrm{i}\frac{\partial}{\partial t}|\psi'(t)\rangle = (d_1(t)H_1 + \cdots + d_l(t)H_l)\,|\psi'(t)\rangle. \tag{26}$$

Thus the complete set of solutions for $|\psi'(t)\rangle$ are $\{|\psi'_n(t)\rangle\}$

$$|\psi'_n(t)\rangle = \mathrm{e}^{-\mathrm{i}\Theta_n(t)}|\phi_n\rangle \tag{27}$$

where

$$\Theta_n(t) = \sum_i n_i \int_0^t \mathrm{d}t\ d_i(t) \tag{28}$$

and $|\phi_n\rangle$ is the common eigenstate of $\{H_i\}$ corresponding to the eigenvalue $n = (n_1, \ldots, n_l)$ which can be obtained from Lie algebraic theory:

$$H_i|\phi_n\rangle = n_i|\phi_n\rangle. \tag{29}$$

From equation (7), we obtain the complete set of solutions of the wavefunction $|\psi(t)\rangle$

$$\begin{aligned} |\psi_n(t)\rangle &= \mathrm{e}^{-\mathrm{i}\Theta_n(t)}U_g^{-1}(t)|\phi_n\rangle \\ &= \sum_m \mathrm{e}^{-\mathrm{i}\Theta_n(t)}D_{mn}(t)|\phi_m\rangle \end{aligned} \tag{30}$$

where $D_{mn}(t)$ is the matrix element of $U_g^{-1}(t)$:

$$D_{mn}(t) = \langle\phi_m|U_1^{-1}(t)U_2^{-1}(t)|\phi_n\rangle. \tag{31}$$

In our specified form of $U_g(t)$, the matrix element $D_{mn}(t)$ can be worked out algebraically. Letting $|0\rangle$ be the lowest-weight state of an irreducible representation of the Lie algebra

$$E_{-\alpha}|0\rangle = 0 \qquad (\alpha = 1, \ldots, M) \tag{32}$$

then $|\phi_n\rangle$ can be written as a product of raising operators acting on the lowest-weight state. For simplicity, let us assume

$$|\phi_n\rangle = c_n E_1^{n_1}E_2^{n_2}\cdots E_M^{n_M}|0\rangle \tag{33}$$

where c_n is a normalization coefficient. Without loss of generality, we assume $E_{-\alpha} = E_\alpha^\dagger$, so that

$$\langle 0|E_\alpha = 0. \tag{34}$$

Thus equation (31) can be written as

$$D_{mn}(t) = c_m^* c_n\langle 0|E_{-M}^{m_M}(t)\cdots E_{-2}^{m_2}(t)E_{-1}^{m_1}(t)E_1^{n_1}(t)E_2^{n_2}(t)\cdots E_M^{n_M}(t)|0\rangle \tag{35}$$

where

$$E_\alpha(t) = U_2^{-1}(t)E_\alpha U_2(t) \tag{36}$$

$$E_{-\alpha}(t) = U_1(t)E_{-\alpha}U_1^{-1}(t). \tag{37}$$

$E_{\pm\alpha}(t)$ can be calculated according to equations (15)–(18), and both are linear functions of $\{H_i, E_{\pm\alpha}\}$. Similarly to the Wick theorem, we can rearrange the order of the H_i, $E_{\pm\alpha}$ in such a way that the E_α ($\alpha = 1, \ldots, M$) appear on the left of the H_i, and the $E_{-\alpha}$ ($\alpha = 1, \ldots, M$) appear on the right of the H_i. Using equations (32) and (34), we obtain $D_{mn}(t)$. In the specified system, the above procedure for calculating $D_{mn}(t)$ can be simplified.

Another immediate result from the above discussion is a set of time-dependent invariant observables which commute with each other [1]:

$$I_i(t) = U_g^{-1}(t) H_i U_g(t). \tag{38}$$

Here $I_i(t)$ is a linear function of the generators of the Lie group and satisfies the equation

$$\mathrm{i}\frac{\partial}{\partial t} I_i(t) + [I_i(t), H(t)] = 0. \tag{39}$$

From this set of time-dependent invariant observables we can also construct the complete set of solutions to the Schrödinger equation [14].

To illustrate the above discussion, consider the example of an SU(3) model describing the interaction between three oscillators in the rotating wave approximation (RWA) [17]. The Hamiltonian reads

$$H = H_0 + H_\mathrm{I} = \sum_{i=1}^{3} x_{ii}(t) a_i^+ a_i + \sum_{i\neq j} x_{ij}(t) a_i^+ a_j \tag{40}$$

where H_0 is three independent oscillators and H_I is the interactions between them; a_i, a_i^+ satisfy the commutation relations $[a_i, a_j^+] = \delta_{ij}$, $[a_i, a_j] = [a_i^+, a_j^+] = 0$; $x_{ij}(t)$ is a non-singular function of time t. The operators

$$A_{ij} = a_i^+ a_j \tag{41}$$

form a basis of U(3) algebra:

$$\left[A_{ij}, A_{kl}\right] = \delta_{jk} A_{il} - \delta_{li} A_{kj}. \tag{42}$$

Since the operator $\sum_{i=1}^{3} a_i^+ a_i$ commutes with every member of the algebra, as well as the Hamiltonian, the algebraic structure of the system is indeed SU(3). This eight-dimensional algebra has two Cartan operators ($A_{11} - A_{22}$, $A_{22} - A_{33}$), three rising operators (A_{12}, A_{13}, A_{23}), and three lowering operators (A_{21}, A_{31}, A_{32}). An irreducible representation space of the algebra is spanned by states

$$\left\{ |\phi(n_1, n_2)\rangle = |n_1, n_2, n - n_1 - n_2\rangle = \frac{1}{\sqrt{n_1! n_2! (n - n_1 - n_2)!}} a_1^{+n_1} a_2^{+n_2} a_3^{+(n-n_1-n_2)} |0\rangle, \quad (0 \leqslant n_1, n_2 \leqslant n) \right\}$$

which are common eigenstates of the Cartan operators, and can be expressed as raising operators acting on the lowest-weight state of the irreducible representation

$$|\phi(n_1, n_2)\rangle = c(n_1, n_2)(A_{13})^{n_1}(A_{23})^{n_2}|0, 0, n\rangle \tag{43}$$

where $c(n_1, n_2)$ is a normalization parameter. The Hilbert space is a summation of all these irreducible representation space.

The gauge transformation that transforms the Hamiltonian into a linear function of the Cartan operators, according the above discussion, is chosen as

$$U_g = \mathrm{e}^{f_{31}A_{31}} \mathrm{e}^{f_{21}A_{21}} \mathrm{e}^{f_{32}A_{32}} \mathrm{e}^{f_{12}A_{12}} \mathrm{e}^{f_{23}A_{23}} \mathrm{e}^{f_{13}A_{13}}. \tag{44}$$

where the time-dependent parameters $f_{ij}(t)$ are determined by the equations

$$
\begin{aligned}
-\mathrm{i}\dot{f}_{13} &= x_{13} - x_{11}f_{13} - x_{31}f_{13}^2 - x_{12}f_{23} + f_{13}(x_{33} - x_{32}f_{23}) \\
-\mathrm{i}\dot{f}_{23} &= x_{23} - x_{22}f_{23} + x_{33}f_{23} - x_{32}f_{23}^2 - f_{13}(x_{21} + x_{31}f_{23}) \\
-\mathrm{i}\dot{f}_{12} &= x_{12} + x_{32}f_{13} - f_{12}(x_{11} + x_{31}f_{13}) - f_{12}^2(x_{21} + x_{31}f_{23}) + f_{12}(x_{22} + x_{32}f_{23}) \\
-\mathrm{i}\dot{f}_{32} &= x_{32} - x_{31}f_{12} - (x_{33} - x_{31}f_{13} - x_{32}f_{23})f_{32} \\
&\quad + (x_{22} + x_{32}f_{23} - f_{12}(x_{21} + x_{31}f_{23}))f_{32} \\
-\mathrm{i}\dot{f}_{21} &= x_{21} + x_{31}f_{23} - f_{21}(x_{22} + x_{32}f_{23} - f_{12}(x_{21} + x_{31}f_{23})) \\
&\quad + f_{21}(x_{11} + x_{31}f_{13} + f_{12}(x_{21} + x_{31}f_{23})) \\
-\mathrm{i}\dot{f}_{31} &= x_{31} - (x_{33} - x_{31}f_{13} - x_{32}f_{23})f_{31} + (x_{11} + x_{31}f_{13} \\
&\quad + f_{12}(x_{21} + x_{31}f_{23}))f_{31} + (x_{21} + x_{31}f_{23})f_{32}.
\end{aligned} \tag{45}
$$

After gauge transformation, the Hamiltonian becomes

$$H = \sum_{i=1}^{3} d_i(t)A_{ii} \tag{46}$$

where

$$
\begin{aligned}
d_1 &= x_{11} + x_{31}f_{13} + f_{12}(x_{21} + x_{31}f_{23}) \\
d_2 &= x_{22} + x_{32}f_{23} - f_{12}(x_{21} + x_{31}f_{23}) \\
d_3 &= x_{33} - x_{31}f_{13} - x_{32}f_{23}.
\end{aligned} \tag{47}
$$

Thus the complete set of solutions to the Schrödinger equation is

$$|\psi_{n_1,n_2}(t)\rangle = \mathrm{e}^{-\mathrm{i}\Theta_n(t)}U_g^{-1}(t)|\phi(n_1, n_2)\rangle \tag{48}$$

$$= \mathrm{e}^{-\mathrm{i}\Theta_n(t)} \sum_{m_1,m_2} D_{m_1,m_2;n_1,n_2}|\phi(m_1, m_2)\rangle \tag{49}$$

where

$$\Theta_n(t) = \int_0^t (n_1d_1(t) + n_2d_2(t) + (n - n_1 - n_2)d_3(t))\ \mathrm{d}t \tag{50}$$

and the matrix elements of $U_g^{-1}(t)$

$$D_{m_1,m_2;n_1,n_2}(f_{ij}) = \langle\phi(m_1, m_2)|U_g^{-1}(t)|\phi(n_1, n_2)\rangle \tag{51}$$

can be calculated as discussed above. It is a function of f_{ij}. This function is independent of the specified system and only depends on the algebraic structure. From direct calculation, we have

$$D_{m_1,m_2;n_1,n_2}(f_{ij}) = c\langle 0, 0, n|(A_{32}(t))^{m_1}(A_{31}(t))^{m_2}(A_{13}(t))^{n_1}(A_{23}(t))^{n_2}|0, 0, n\rangle \tag{52}$$

where

$$
\begin{aligned}
&c = c(m_1, m_2)c(n_1, n2) \\
&A_{13}(t) = A_{13} - A_{23}f_{21} + A_{11}f_{31} - A_{21}f_{21}f_{31} + A_{12}f_{32} - A_{22}f_{21}f_{32} \\
&\qquad\qquad + A_{33}(-f_{31} + f_{21}f_{32}) + A_{31}(-f_{31}^2 + f_{21}f_{31}f_{32}) + A_{32}(-(f_{31}f_{32}) + f_{21}f_{32}^2) \\
&A_{23}(t) = A_{23} + A_{21}f_{31} + A_{22}f_{32} - A_{33}f_{32} - A_{31}f_{31}f_{32} - A_{32}f_{32}^2 \\
&A_{31}(t) = A_{31} - A_{32}f_{12} + A_{11}f_{13} - A_{12}f_{12}f_{13} + A_{21}f_{23} - A_{22}f_{12}f_{23} \\
&\qquad\qquad + A_{33}(-f_{13} + f_{12}f_{23}) + A_{13}(-f_{13}^2 + f_{12}f_{13}f_{23}) + A_{23}(-(f_{13}f_{23}) + f_{12}f_{23}^2) \\
&A_{32}(t) = A_{32} + A_{12}f_{13} + A_{22}f_{23} - A_{33}f_{23} - A_{13}f_{13}f_{23} - A_{23}f_{23}^2.
\end{aligned}
\tag{53}
$$

Substituting equations (53) in (52) and replacing A_{ij} and $|0, 0, n\rangle$ by $a_i^+ a_j$ and $(1/\sqrt{n!})a_3^{+n}|0, 0, 0\rangle$, respectively, $D_{m_1,m_2;n_1,n_2}$ can readily be worked out by means of the Wick theorem.

In summary, we have shown a unified way to obtain the exact solutions of the Schrödinger equation for a non-autonomous quantum system whose Hamiltonian is a linear function of the generators of a semisimple Lie group. The key step is to chose a gauge transformation that transforms the Hamiltonian into a linear function of the Cartan operators. In the specified form, the matrix elements of the gauge transformation can be calculated algebraically. In fact, the above method can also be applied to some non-semisimple Lie algebras, such as the h(4) and U(N) algebras. The U(N) algebra can also be treated as the subalgebra of Sp$(2N)$, which is semisimple. Indeed, if the algebra considered has a basis in the form of (2) with commutation relations in the form of (3), the above method can be used.

In general, we cannot obtain analytic results by this method. To determine the time-dependent coefficients of the gauge transformation, one needs to solve a set ordinary differential equations like (21), (24) and (45). Thus, this method can be viewed as transforming the Schrödinger equation, a partial differential equation, into a set of ordinary differential equations. These ordinary differential equations need to be solved numerically in general. However, it is easier to obtain numerical solutions of ordinary equations than that of a partial differential equation. Furthermore, to solve these ordinary differential equations, one does not need the ‘cut-off’ approximation (choosing a finite dimensional subspace of the Hilbert space to solve the Schrödinger equation). In this sense, this kind of solution is ‘exact’.

This method has a limitation. To obtain the matrix elements of the evolution operator, this method needs a lowest- (or highest-)weight state $|0\rangle$, so that any lowering (or raising) operator acting on it is equal to zero. When there is no such state in the state space of the system under consideration, this method does not work.

Note that $\{|\phi_n\rangle\}$ in (33) is not the general form of a weight vector. It should be written as a more complicatedly ordered product of raising operators acting on $|0\rangle$. This case can be treated in the same way as above.

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References

[1] Wang S J, Li F L and Weiguny A 1993 *Phys. Lett.* **180A** 189
[2] Paul W 1990 *Rev. Mod. Phys.* **62** 531
Brown L S 1991 *Phys. Rev. Lett.* **66** 527
[3] Wang S J, Zuo W, Weiguny A and Li F L 1994 *Phys. Lett.* **196A** 7
Wang S J and Zuo W 1994 *Phys. Lett.* **196A** 13
Zuo W and Wang S J 1995 *Acta Phys. Sinica* **44** 1354
Zuo W and Wang S J 1995 *Acta Phys. Sinica* **44** 1363
[4] Shore B W and Knight P L 1993 *J. Mod. Opt.* **40** 1195
[5] Yu S, Rauch H and Zhang Y 1995 *Phys. Rev.* A **52** 2585
[6] Kibler M and Negadi T 1980 *Lett. Nuovo Cimento* **37** 225
Cornish F H 1984 *J. Phys. A: Math. Gen.* **17** 323
Chen A C and Kibler M 1985 *Phys. Rev.* A **31** 3960
[7] Xu B W and Zeng Q 1991 *Acta Phys. Sinica* **40** 1212
Xu B W and Gu W H 1993 *Acta Phys. Sinica* **42** 1050
[8] Wei J and Norman E 1963 *J. Math. Phys.* **4** 575
[9] Shi S and Rabitz H 1988 *J. Chem. Phys.* **88** 7508
[10] Gilmore R and Yuan J M 1987 *J. Chem. Phys.* **86** 130
Gilmore R and Yuan J M 1989 *J. Chem. Phys.* **91** 917
[11] Recamier J, Micha D A and Gazdy B 1985 *Chem. Phys. Lett.* **119** 383
Recamier J, Micha D A and Gazdy B 1986 *J. Chem. Phys.* **85** 5093
[12] Benjamin I 1986 *J. Chem. Phys.* **85** 5611
[13] Gilmore R 1974 *Lie Groups, Lie Algebras and Some of Their Applications* (New York: Wiley)
Cheng J Q 1989 *Group Representation Theory for Physicists* (Singapore: World Scientific)
[14] Lewis H R 1969 *J. Math. Phys.* **10** 1458
[15] Singh S 1982 *Phys. Rev.* A **25** 3206
[16] Bonatsos D, Daskaloyannis C and Lalazissis G A 1993 *Phys. Rev.* A **47** 3448
[17] Khidekel V 1995 *Phys. Rev.* E **52** 2510